Taking the formula $\frac{v^2}{\mu} = \frac{2}{r} - \frac{1}{a}$, let r be the radius of the sphere we have been considering, *i.e.* $\log r = 8{\cdot}03865$. Let us in each case assume that the comet enters the sphere with parabolic velocity. If its period on emergence is 772 years (Comet 1882 II.), log $a = 1{\cdot}9251$, and the loss of velocity is 1 in 30800.

If its period is 240 years (assumed minimum value for 1680 Comet), the loss of velocity is 1 in 14100.

If its period is 77 years (assumed minimum value for 1843 I.) the loss of velocity is 1 in 6624.

While for 40 and 33 years (minimum values for 1880 I. and 1887 I.), the losses are 1 in 4280 and 1 in 3764 respectively.

Thus we can assert with moral certainty that in no case was the diminution of velocity as great as 1 in 4000, while in the one case where the period is determined with fair accuracy by the observations, we obtain 1 in 30,000 as the limit.

The 1882 Comet was accurately observed for ten days before perihelion, and the observations made then fit on quite satisfactorily with the orbit deduced from post-perihelion observations (*M.N.*, **80**, 409), thus affording an independent proof of the negligible nature of the resistance.

On some Remarkable Properties of Diurnal Motion.
By N. Liapin.

1. In 1853 Prestel pointed out* an exceedingly simple method of finding latitude. Differentiating the formula

$$\cos z = \sin \phi \sin \delta + \cos \phi \cos \delta \cos t,$$

we have in general

$$\frac{dz}{dt} = \cos \delta \sin q = \cos \phi \sin a$$

(q denoting the parallactic angle).

In the prime vertical $a = \pm 90°$, whence

$$\left(\frac{dz}{dt}\right)_{p.v.} = \pm \cos \phi \quad . \quad . \quad . \quad . \quad (1)$$

When looking for other differential relations of this type, I found that there exist two systems of similar formulæ, which determine both latitude and declination. Thus for stars at elongation $q = \pm 90°$, whence

$$\left(\frac{dz}{dt}\right)_e = \pm \cos \delta \quad . \quad . \quad . \quad . \quad (2)$$

* Prestel, "Höchst einfaches Verfahren die geographische Breite zu bestimmen," *A.N.*, **37**, 281. See also W. Chauvenet, *Spherical and Practical Astronomy*, 1868, **1**, 303.

Differentiating the formula

$$\sin z \cos a = -\cos \phi \sin \delta + \sin \phi \cos \delta \cos t,$$

in the case of azimuth we get in general

$$\frac{da}{dt} = \frac{\cos \delta \cos q}{\sin z}.$$

In the prime vertical $\cos q = \sin \phi \sin z \sec \delta$, at elongation $q = \pm 90°$, in horizon $z = 90°$, $\cos q = \sin \phi \sec \delta$, whence *

$$\left(\frac{da}{dt}\right)_{p.v.} = \sin \phi \quad (3); \quad \left(\frac{da}{dt}\right)_e = 0 \quad (4); \quad \left(\frac{da}{dt}\right)_h = +\sin \phi \quad (5)$$

and therefore

$$\left(\frac{da}{dt}\right)_{p.v.} = \left(\frac{da}{dt}\right)_h.$$

(The suffix h denotes the horizon.)

On dividing (3) by (1) we have

$$\left(\frac{da}{dz}\right)_{p.v.} = \pm \tan \phi \quad . \quad . \quad . \quad . \quad (6)$$

On the other hand, from the general formula

$$\frac{da}{dz} = \frac{\cot q}{\sin z}$$

we have

$$\left(\frac{da}{dz}\right)_e = 0 \quad (7); \quad \left(\frac{da}{dz}\right)_h = \cot q_h \quad (8).$$

2. A system of similar relations may be obtained also in the case of parallactic angle. Differentiating the formula

$$\sin z \cos q = \sin \phi \cos \delta - \cos \phi \sin \delta \cos t,$$

we have in general

$$\frac{dq}{dt} = \frac{\cos \phi \cos a}{\sin z}.$$

In the prime vertical $a = \pm 90°$, at elongation $\frac{\cos a}{\sin z} = -\sin \delta \sec \phi$, in horizon $z = 90°$, $\cos a = \sin \delta \sec \phi$, whence

$$\left(\frac{dq}{dt}\right)_{p.v.} = 0 \quad (9); \quad \left(\frac{dq}{dt}\right)_e = -\sin \delta \quad (10); \quad \left(\frac{dq}{dt}\right)_h = -\sin \delta \quad (11)$$

and therefore

$$\left(\frac{dq}{dt}\right)_e = \left(\frac{dq}{dt}\right)_h.$$

* Formulæ (3) and (5) are given first by W. Struve in his *Breitengradmessung in den Ostseeprovinzen Russlands*, 1831, 1, 104.

On dividing (10) by (2) we have

$$\left(\frac{dq}{dz}\right)_e = \mp \tan \delta \quad . \quad . \quad . \quad . \quad (12)$$

On the other hand, from the general formula

$$\frac{dq}{dz} = \frac{\cot a}{\sin z}$$

we have

$$\left(\frac{dq}{dz}\right)_{p.v.} = 0 \quad . \quad . \quad (13); \qquad \left(\frac{dq}{dz}\right)_h = \cot a_h \quad . \quad . \quad (14)$$

For the horizon we have, further,

$$\left(\frac{dz}{dt}\right)_h = \cos \delta \sin q_h = \cos \phi \sin a_h \quad . \quad . \quad (15)$$

3. The values of the fifteen differential coefficients thus obtained are summarised for convenience in the following table:—*

$$\left(\frac{dz}{dt}\right)_{p.v.} = \pm \cos \phi, \qquad \left(\frac{dz}{dt}\right)_e = \pm \cos \delta, \qquad \left(\frac{dz}{dt}\right)_h = \frac{\cos \delta \sin q_h}{\cos \phi \sin a_h}$$

$$\left(\frac{da}{dt}\right)_{p.v.} = \sin \phi, \qquad \left(\frac{da}{dt}\right)_e = 0, \qquad \left(\frac{da}{dt}\right)_h = + \sin \phi$$

$$\left(\frac{dq}{dt}\right)_{p.v.} = 0, \qquad \left(\frac{dq}{dt}\right)_e = - \sin \delta, \qquad \left(\frac{dq}{dt}\right)_h = - \sin \delta$$

$$\left(\frac{da}{dz}\right)_{p.v.} = \pm \tan \phi \qquad \left(\frac{da}{dz}\right)_e = 0, \qquad \left(\frac{da}{dz}\right)_h = \cot q_h$$

$$\left(\frac{dq}{dz}\right)_{p.v.} = 0, \qquad \left(\frac{dq}{dz}\right)_e = \mp \tan \delta, \qquad \left(\frac{dq}{dz}\right)_h = \cot a_h .$$

Regarding the above system of first differential coefficients as a fundamental one, we proceed now to establish the simplest relations which exist between the second differential coefficients.

4. Calculating the general expressions of second differential coefficients of zenith distance, azimuth, and parallactic angle, we obtain:—

$$\frac{d^2 z}{dt^2} = \frac{dq}{dt} \cos \delta \cos q$$

$$\frac{d^2 a}{dt^2} = - \frac{\cos \delta}{\sin^2 z}\left(\sin z \sin q \frac{dq}{dt} + \cos q \cos z \frac{dz}{dt}\right)$$

$$\frac{d^2 q}{dt^2} = - \frac{\cos \phi}{\sin^2 z}\left(\sin z \sin a \frac{da}{dt} + \cos a \cos z \frac{dz}{dt}\right)$$

$$\frac{d^2 a}{dz^2} = - \frac{1}{\sin z}\left(\frac{1}{\sin^2 q} \frac{dq}{dz} + \cot q \cot z\right)$$

$$\frac{d^2 q}{dz^2} = - \frac{1}{\sin z}\left(\frac{1}{\sin^2 a} \frac{da}{dz} + \cot a \cot z\right).$$

* The upper sign applies to a west star, the lower to an east star.

The following table, which is quite similar to the preceding one, contains fifteen values of these coefficients for the prime vertical, elongation, and horizon:—

$$\left(\frac{d^2z}{dt^2}\right)_{p.v.}=0,\qquad \left(\frac{d^2z}{dt^2}\right)_e=0,\qquad \left(\frac{d^2z}{dt^2}\right)_h=-\sin\phi\sin\delta$$

$$\left(\frac{d^2a}{dt^2}\right)_{p.v.}=\mp\frac{\cos\phi\sin\delta}{\sin z_{p.v.}},\qquad \left(\frac{d^2a}{dt^2}\right)_e=\pm\frac{\sin\delta\cos\delta}{\sin z_e},\qquad \left(\frac{d^2a}{dt^2}\right)_h=\sin\delta\cos\delta\sin q_h$$

$$\left(\frac{d^2q}{dt^2}\right)_{p.v.}=\mp\frac{\cos\phi\sin\phi}{\sin z_{p.v.}},\qquad \left(\frac{d^2q}{dt^2}\right)_e=\pm\frac{\cos\delta\sin\phi}{\sin z_e},\qquad \left(\frac{d^2q}{dt^2}\right)_h=-\sin\phi\cos\phi\sin$$

$$\left(\frac{d^2a}{dz^2}\right)_{p.v.}=\mp\frac{\tan\phi}{\tan z_{p.v.}},\qquad \left(\frac{d^2a}{dz^2}\right)_e=\pm\frac{\tan\delta}{\sin z_e},\qquad \left(\frac{d^2a}{dz^2}\right)_h=-\frac{\cot a_h}{\sin^2 q_h}$$

$$\left(\frac{d^2q}{dz^2}\right)_{p.v.}=\mp\frac{\tan\phi}{\sin z_{p.v.}},\qquad \left(\frac{d^2q}{dz^2}\right)_e=\pm\frac{\tan\delta}{\tan z_e},\qquad \left(\frac{d^2q}{dz^2}\right)_h=-\frac{\cot q_h}{\sin^2 a_h}.$$

The following simple relations may be easily derived:—

$$\left(\frac{d^2q/dt^2}{d^2q/dz^2}\right)_{p.v.}=\cos^2\phi,\quad \left(\frac{d^2a/dt^2}{d^2q/dt^2}\right)_{p.v.}=\cos z_{p.v.},\quad \left(\frac{d^2q/dt^2}{d^2z/dt^2}\right)_h=-\tan a_h$$

$$\left(\frac{d^2a/dt^2}{d^2a/dz^2}\right)_e=\cos^2\delta,\quad \left(\frac{d^2q/dt^2}{d^2a/dt^2}\right)_e=\cos z_e,\quad \left(\frac{d^2a/dt^2}{d^2z/dt^2}\right)_h=-\tan q_h.$$

The following simple relations between the first and second differential coefficients in the prime vertical and at elongation

$$\left(\frac{da/dz}{d^2q/dz^2}\right)_{p.v.}=-\sin z_{p.v.},\qquad \left(\frac{dq/dz}{d^2a/dz^2}\right)_e=-\sin z_e$$

$$\left(\frac{da/dz}{d^2a/dz^2}\right)_{p.v.}=-\tan z_{p.v.},\qquad \left(\frac{dq/dz}{d^2q/dz^2}\right)_e=-\tan z_e$$

give

$$\left\{\frac{(d^2q/dz^2)(d^2a/dt^2)}{d^2q/dt^2}\right\}_{p.v.}=\left(\frac{d^2a}{dz^2}\right)_{p.v.}\qquad \left\{\frac{(d^2a/dz^2)(d^2q/dt^2)}{d^2a/dt^2}\right\}_e=\left(\frac{d^2q}{dz^2}\right)_e.$$

On the horizon:

$$\left(\frac{d^2z/dt^2}{dq/dt}\right)_h=\sin\phi,\quad \left(\frac{d^2q/dt^2}{dz/dt}\right)_h=-\sin\phi,\quad \left(\frac{da/dz}{d^2q/dz^2}\right)_h=-\sin^2 a_h$$

$$\left(\frac{d^2a/dt^2}{dz/dt}\right)_h=\sin\delta,\quad \left(\frac{d^2z/dt^2}{da/dt}\right)_h=-\sin\delta,\quad \left(\frac{dq/dz}{d^2a/dz^2}\right)_h=-\sin^2 q_h.$$

From all these formulæ there clearly stands out the remarkable parallelism between four elements—latitude, declination, azimuth, and parallactic angle. In fact, for stars culminating south of the zenith, latitude and azimuth analytically play the same part as declination and parallactic angle for stars culminating between the pole and the zenith. Azimuth of stars of the first group varies from 0° to 360° as reckoned from upper culmination in the

direction of diurnal motion, whilst parallactic angle for these stars varies from 0° to 0°, having a maximum and a minimum. Parallactic angle of stars of the second group varies from 0° to 360° as reckoned from lower culmination in the direction inverse to that of diurnal motion, whilst azimuth of these stars varies from 180° to 180°, having a minimum and a maximum. Hence arise different signs of all first and second differential coefficients in the prime vertical and elongation, as may be seen from our two tables. Parallactic angle of stars of the first group becomes a maximum or a minimum when their azimuth is 90° or 270° respectively, whereas azimuth of stars of the second group becomes a maximum or a minimum when its parallactic angle is 270° or 90° respectively. Moreover, it is a matter of interest that first differential coefficients of azimuth in the prime vertical define latitude only, those of parallactic angle at elongation declination only. First differential coefficients of azimuth at elongation and of parallactic angle in the prime vertical vanish. In the horizon first differential coefficients of azimuth define latitude and parallactic angle, whilst those of parallactic angle define declination and azimuth.

5. From the inspection of our formulæ two important properties of latitude and azimuth on the one hand, declination and parallactic angle on the other, become obvious, viz. *reciprocity and conjugation.* Thus, our analysis gives a full account of all the circumstances of diurnal motion, thus drawing an interesting law of symmetry between these elements, which at first appearance seem to have no internal connection with each other.

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On the Suggested Increase in Period of Variable Stars in Phillips's Group I., with Particular Notes on U Virginis, S Cephei, T Cassiopeiæ, S Ursæ Majoris, R Serpentis, T Hydræ, T Herculis, and other Stars. By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. In a former paper the exceptional stars of Group II. were shown to have decreasing periods: in the present a corresponding increase of period for some Group I. stars is indicated. The word "exceptional" may be emphasised. The examination of all the stars for which we have light curves good enough to assign the group is not yet completed: but it has gone far enough to show that for the stars in general we have not as yet a long enough series of observations to be sure of the changes in period, which are usually small. We can only note that for the stars which show large changes the evidence seems to point all one way.